AD-A166 055 SIEVES FOR GAUSSIAN PROCESSES(U) WISCONSIN UNIV-MILWAUKEE DEPT OF MATHEMATICAL SCIENCES J H BEDER 27 NOV 85 AFOSR-TR-86-0069 AFOSR-84-0329 1/1

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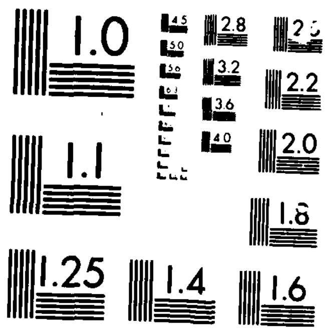

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Sieves for Gaussian Processes

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2,3. Research objectives, status of work:

Many phenomena can be modelled as signals to which noise has been added. Frequently, it is assumed that the noise is governed by the so-called Gaussian probability law. If the signal itself is non-random, the most important questions are extraction of the signal and description of the noise. Statistically these are the problems of estimating the mean and covariance, respectively, of a Gaussian process. A classical statistical method, maximum likelihood estimation, breaks down when the signal is "infinite-dimensional," which is the case in many applications. The method of sieves often provides reasonable estimators in such circumstances, and the PI has introduced sieve estimators for the mean and covariance of a Gaussian process. This project is aimed in part at investigating the properties of these estimators, with no assumptions on the "time" parameter (it is only assumed to be an element of a set T).

A sieve estimator is doubly indexed. The first index, say n, is the sample size - in this case, the number of realizations of the process that one observes. The second, say d, is a sieve parameter. Both these indices are allowed to increase without bound to obtain certain asymptotic properties of the estimator; one typically requires that d go to infinity at some rate with respect to n. For a more complete account of sieve estimation, see Grenander [1981] or Geman and Hwang [1982].

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The first goal of this project is to develop the properties of a new sieve estimator for the mean, say $m(t)$, of a Gaussian process when the covariance is known. Let $\mathcal{H}$ be the reproducing kernel Hilbert space of the covariance function; then $\mathcal{H}$ is the set of all possible mean functions of the process. Consider the following condition:

(S) the true mean function belongs to a known separable subspace $\mathcal{H}_0$ of $\mathcal{H}$.

(If $\mathcal{H}$ is separable, we take $\mathcal{H}_0 = \mathcal{H}$.) The sieve estimator, say $\hat{m}(t)$, is defined in terms of a complete orthonormal basis of $\mathcal{H}_0$; a precise definition of $\hat{m}$ is given in the project proposal. The PI has now shown the following:

a. For fixed n and d, the estimator $\hat{m}(t)$ is a Gaussian process. When (S) holds, one can give a simple formula for the mean and covariance of $\hat{m}(t)$, from which one can show that, at each t, $\hat{m}(t)$ is asymptotically unbiased as $d \to \infty$ and is weakly consistent for $m(t)$ if $d = O(n)$.

b. Both $\hat{m}$ and m are elements of $\mathcal{H}$, and using the norm of $\mathcal{H}$ as a metric one can show that the distance between $\hat{m}$ and m goes to zero almost surely under two assumptions: (i) that (S) holds, and (ii) that $d \to \infty$ and $d/n \to 0$ as $n \to \infty$. This is "best possible" in the sense that if $d/n \to c > 0$, then one cannot even get weak convergence of the distance to zero.

The second part of this project involves the estimation of the covariance, say R, of a Gaussian process whose mean is known (and may be assumed to be identically zero). In the project proposal the PI conjectured: when the Hilbert space $\mathcal{H}$ above is infinite dimensional, the likelihood function of the covariance is unbounded, so that the maximum likelihood estimator (MLE) fails to exist.

c. The above conjecture has been proved. In particular, consider this condition:

(C) The true covariance is specified by a bivariate expansion using a fixed countable orthonormal set in $\mathcal{H}$.

The PI has shown that the MLE exists almost surely if the set in (C) is finite, and fails to exist (the likelihood being unbounded) almost surely if the set of infinite.

Thus a sieve estimator is necessary in general, and the "finite" case above actually leads to a natural sieve estimator $\hat{R}$, with dimension d as sieve parameter. The PI has established the following.

d. When (C) holds, the estimator $\hat{R}$, viewed as a stochastic process on TxT, has a simple distribution in terms of independent χ^2 random variables, from which it follows that, at each $(s,t) \in T \times T$, $\hat{R}(s,t)$ is asymptotically unbiased as $d \to \infty$ and is weakly consistent for $R(s,t)$ if $d = O(n)$.

The coefficients in the expansion in (C) form a square-summable sequence, so we may reparametrize the model by a subset of an ℓ^2-space. Using the metric defined by the ℓ^2-norm, the PI has shown the following:

e. If $d \to \infty$ and $d/n \to 0$, then we have strong consistency in ℓ^2, under assumption (C).

f. The metric in ℓ^2 is equivalent to a natural distance between covariances given by the norm in $\mathcal{H} \odot \mathcal{H}$, the second symmetric tensor power of $\mathcal{H}$.

Aside from further investigation of these sieve estimates, the next goal of the project is to weaken or remove a restriction of continuity placed on a filter for a stochastic Gaussian signal invented by M. Driscoll. Details are given in the PI's project description.

References

Geman, S., and Hwang, C.-R. (1982). Nonparametric maximum likelihood estimation by the method of sieves, Ann. Statist. 10: 401-414.

Grenander, U. (1981). Abstract Inference. Wiley, New York.

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4. Written Publications:

Jay H. Beder, "A sieve estimator for the mean of a Gaussian process," Annals of Statistics (submitted).

Jay H. Beder, "A sieve estimator for the covariance of a Gaussian process," Annals of Statistics (in preparation).

5. Professional Personnel:

None besides the PI.

6. Interactions:

(a) Papers presented:

(i) "Sieve estimation for Gaussian processes," Joint Statistical Meetings, Las Vegas, Nevada, August 7, 1985.

(ii) "Sieve estimation for Gaussian processes," International Symposium on Foundations of Statistical Inference," Tel Aviv, Israel, December 15-19, 1985.

(b) Consultative and advisory functions: none.

7. New discoveries, inventions, patents: none.

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The PI has established several important properties of a proposed sieve estimator for the mean of a Gaussian process of known convariance: The estimator is itself a Gaussian process; under a separability assumption, it is asymptotically unbiased and weakly consistent at each "time" t, and is strongly consistent (globally) in an appropriate norm.

For a Gaussian process with zero mean and unknown convariance, the PI has shown that the likelihood for the covariance is in general unbounded almost surely. Moreover, he has developed properties of a proposed sieve estimator for the covariance analogous to those for the mean. No assumption is made about the nature of the "time" parameter t, either for the mean estimator or for the covariance estimator.

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